2020 Power Team
Texas A&M High School Mathematics Contest
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In the following series of problems there is one or several particles of some masses on the real line at any moment t of time (time takes only integer values). If at moment t there is a particle of mass m at coordinate x, then (except for some special cases mentioned in the problems) the next moment $t+1$ it splits into two particles of mass $m/2$ each: one at coordinate $x+1$, and one at coordinate $x-1$ (all particles split simultaneously). If two particles of masses m_1 and m_2 meet at the same point of the line, then they merge and we get one particle at that point of mass m_1+m_2.

For example, if at moment t there are particles of masses m_1, m_2, m_3, m_4 at coordinates $1, 2, 3, 4$, respectively, and no particles at other coordinates, then the next moment there are particles of masses $m_1/2, m_2/2, (m_1+m_3)/2, (m_2+m_4)/2, m_3/2, m_4/2$ at coordinates $0, 1, 2, 3, 4, 5$, respectively.

Problem 1. Suppose that at the initial moment $t=0$ we have one particle of mass 1 at coordinate 0. Find the masses and coordinates of all particles at the moment $t>0$.

Problem 2. How will the answer to Problem 1 change if we have an absorbing screen at coordinate $k>0$: every particle that reaches that point is annihilated.

Problem 3. Find the answer when there are two absorbing screens at coordinates $k>0$ and $l<0$.

Problem 4. What if we have a reflective screen at coordinate $k>0$: if a particle is at coordinate k at moment t, then it doesn't split into two particles, but moves instead to coordinate $k-1$ at moment $t+1$ without changing its mass.

Problem 5. Suppose now that we have a semi-transparent membrane at coordinate $k>0$: if a particle of mass m is at coordinate k at moment t, then it is split into two particles next moment $t+1$: a particle of mass pm at coordinate $k-1$ and a particle of mass qm at coordinate $k+1$, where p, q are positive constants such that $p+q=1$.

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In the following problems, we are looking for configurations of points inside a square room that stay as far from one another as possible.

Let Q be a unit square in the Euclidean plane (that is, a square with sides of length 1). Suppose S is a finite set of points inside the square Q (some points may lie on the boundary of Q). We denote by $sd(S)$ the minimal distance between distinct points in the set S. For any integer $n \geq 2$, let d_n be the maximal value of $sd(S)$ over all sets S of n points. A set S of n points inside the square Q is called an **optimal configuration** if $sd(S) = d_n$.

Problem 6. Find the optimal configurations of n points and find d_n for $n = 2, 4$, and 5. Prove that they are optimal.

Problem 7. The same for $n = 3$.

Problem 8. Find d_n and an optimal configurations of $n = 6$ and 8 points. (A rigorous proof is not required.)

Problem 9. Introduce Cartesian coordinates such that the vertices of the square are $(0,0), (1,0), (0,1)$, and $(1,1)$. For $k \geq 1$, let S_k be the set of points with coordinates of the form $\left(\frac{m_1}{k}, \frac{m_2}{k}\right)$ for $0 \leq m_1, m_2 \leq k$. Prove that for all sufficiently large k the configuration S_k is not optimal.