Applied Analysis Qualifier Syllabus

12/2020

Function spaces and operators.

- Banach and Hilbert spaces
- L^2 spaces
- Sobolev spaces
- C^k and sequence spaces
- Riesz Representation Theorem

Approximation Analysis.

- Completeness of orthonormal expansions
- Fourier series
- FFT
- Weierstrass approximation theorem
- Splines

Compact operators.

- Finite rank and Hilbert-Schmidt operators
- Spectral theory
- Integral equations
- Contraction mapping theorem & Neumann series

Differential Operators.

- Distributions
- Green's functions
- Sturm-Liouville problems
- Courant-Fischer minimax theorem

References.

James P. Keener: Principles of Applied Mathematics: Transformation and Approximation second (revised) edition, Westview Press, Boulder, Colorado, 2000.

G. P. Tolstov: Fourier Series, Dover Publications, New York, 1976.

H. J. Wilcox and D. L. Meyers: An Introduction to Lebesgue Integration and Fourier Series, Dover Publications, New York, 1994.

F. Riesz and B. Sz.-Nagy: Dover Publications, New York, 1990.

PROPOSED CONTENT OF THE QUALIFYING EXAM ON APPL. MATH/NUMERICAL ANALYSIS. Part II. Numerical Analysis

Completed by Jean-Luc Guermond, Raytcho Lazarov and Joseph Pasciak,
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Content

(1) Finite element method
(2) Numerical methods for parabolic equations
(3) References

1. Finite element method

(Johnson, Ciarlet, Strang & Fix, Ern & Guermond, Grossmann et al)

(1) Weak (variational) formulation of second order elliptic problems and characterization of the energy space: essential and natural boundary condition.
(2) Ritz-Galerkin method and finite element method.
(3) Finite element spaces of piece-wise linear and quadratic polynomials (over triangles and tetrahedra) and piece-wise bilinear and biquadratic polynomials over rectangles.
(4) Error estimates, Bramble-Hilbert lemma, Nitsche trick. Strans's Lemmas.
(5) Variational "crimes": nonconforming spaces, and approximation of the bilinear and linear forms by quadrature rules.
(6) Galerkin finite element method for transient problems.

2. Numerical methods for parabolic problems

(Johnson, Larsen &Thomee)

(1) Finite difference approximations in time: explicit, implicit and Crank-Nicolson schemes, multistep methods, Runge-Kutta methods.
(2) Stability: maximum principle, Fourier mode analysis, matrix stability and energy type estimates (Courant condition).
(3) Error estimates for finite element and finite difference approximations.

3. References

(1) S. Larsen and V. Thomee, Partial Differential Equations with Numerical Methods, Springer-Verlag, Texts in Applied Mathematics 45, 2003
(2) C. Johnson, Numerical Solutions of PDE's by the Finite Element Method, Cambridge University Press, 1987.
(3) C. Grossmann, H.-G. Roos, and M. Stynes, Numerical Treatment of PDEs, Springer, 2007.
(4) A. Ern and J-L Guermond, Theory and Practice of Finite Elements, Springer, 2004.
(5) Ph. Ciarlet, The Finite Element Method for Elliptic Problems, SIAM, 2002.
(6) G. Strang and G. Fix, An Analysis of the Finite Element Method, Prentice Hall, Englewood Cliffs, N.J., 1973.