Texas A&M Mathematics Department

REU Summer Research Conference, Wednesday July 23, 2014

Blocker 117

9:45 - 9:55 Jean Marie Linhart: Opening Remarks

Session 1 Moderator: J. Maurice Rojas

10:00 - 10:15 Joann M. Coronado: *Visualizing the $\mathcal{A}$-Discriminant Variety and their Tropicalizations*

10:20 - 10:35 Samuel Pérez-Ayala: *Visualizing $\mathcal{A}$-Discriminant Chambers*

10:40 - 10:55 Bithiah Yuan: *Computing the Tropical $\mathcal{A}$-Discriminant*

10 minute break

Session 2 Moderator: Jean Marie Linhart

11:05 - 11:20 Meredith McCormack-Mager: *Inoculation Strategies for Polio: Modeling the Effects of a Growing Population on Public Health Outcomes*

11:25 - 11:40 Paula Burkhardt: *Strong Solution to Smale's 17th Problem for Strongly Sparse Systems*

11:45 - 12:00 Caleb Xavier Bugg: *Binomial Solutions to Smale's 17th Problem and their Application to Chemical Reaction Networks*

Lunch 12 - 1 pm

Session 3 Moderator: Riad Masri

1:00 - 1:15 Robert Cass: *The Chowla-Selberg Formula for Quartic Abelian CM fields*[1]

1:20 - 1:35 Taylor McClanahan: *Intervention Model for Malaria*

1:40 - 1:55 Oscar González: *Zeros of Maass Forms*

10 minute break

Session 4 Moderator: Jay Walton

2:05 - 2:20 Edna Jones: *Representations by Ternary Quadratic Forms*

2:25 - 2:40 Matea Alvarado: *Modeling Cyclophosphamide's Effect on Leukocytes*

2:45 - 3:00 Jane Thompson: *A Class Structured Mathematical Model For Polio Virus In Nigeria*

10 minute break

Session 5 Moderator: Matt Young

3:10 - 3:25 Katy Weber: *Effective Non-vanishing of Class Group L-functions for Biquadratic CM Fields (part 1)*

3:30 - 3:45 Emily Pierce: *Effective Non-vanishing of Class Group L-functions for Biquadratic CM Fields (part 2)*

3:50 - 4:00 Paulo Lima-Filho: Closing Remarks

[1] complex multiplication fields

Titles and Abstracts

10:00 - 10:15 *Visualizing the $\mathcal{A}$-Discriminant Variety and their Tropicalizations*
Joann M. Coronado, Texas A&M University, Corpus Christi
Research Mentor: J. Maurice Rojas, Alperen Ergur, Kaitlyn Phillipson

The study of polynomial systems has captivated mathematicians for over 2000 years and evolved into the subject of algebraic geometry. In the 20th century, new and deep connections to combinatorics and algorithmic complexity were established and continue to develop. A more recent point of view to understanding polynomials is by working more closely with their set of exponent vectors $\mathcal{A}$, instead of just using their classification by degree (quadratic, cubic, quartic, etc.).

For each $\mathcal{A}$, we can study the topology of the possible real zero sets by first looking at the polynomials with singular zero sets. This defines the $\mathcal{A}$-discriminant variety $\nabla_{\mathcal{A}}$. The connected components of the complement of $\nabla_{\mathcal{A}}$ are called *discriminant chambers*, and are actually regions in coefficient space where the topology of the real zero set is constant. For this reason, visualizing discriminant chambers, and approximating them, is important. In particular, approximating complicated semi-algebraic sets like chambers can be dramatically sped up by suitable polyhedral approximations, such as the tropical discriminant.

We discuss the applications of discriminant chambers to algorithmic algebraic geometry as well as to other areas.

10:20 - 10:35 *Visualizing $\mathcal{A}$-Discriminant Chambers*
Samuel Pérez-Ayala, University of Puerto Rico, Río Piedras Campus
Research Mentor: J. Maurice Rojas, Alperen Ergur, Kaitlyn Phillipson

For any finite set $\mathcal{A} \subset \mathbb{Z}^n$, consider the family $\mathcal{F}_{\mathcal{A}}$ of polynomials of the form $f(x) = \sum_{a \in \mathbb{A}} c_a x^a$. The $\mathcal{A}$-discriminant is the unique (up to sign) irreducible polynomial $\Delta_{\mathcal{A}} \in \mathbb{Z}[c_a \mid a \in \mathbb{A}] \setminus \{0\}$ satisfying $\Delta_{\mathcal{A}}(c_a \mid a \in \mathbb{A}) = 0$ when f has a degenerate root in $(\mathbb{C}^*)^n$. It is well-known, via classical Morse Theory, that the real zero set Z of $\Delta_{\mathcal{A}}$ partitions $\mathcal{F}_{\mathcal{A}}$ into chambers where the topology of the real zero set of f is constant. While $\Delta_{\mathcal{A}}$ is hard to compute, we nevertheless have a way to visualize its real zero set. We are developing a software package for visualizing $\mathcal{A}$-discriminant chambers when $\mathcal{A}$ has cardinality $n + 4$.

10:40 - 10:55 *Computing the Tropical $\mathcal{A}$-Discriminant*
Bithiah Yuan, University of Hawaii at Hilo
Research Mentor: J. Maurice Rojas, Alperen Ergur, Kaitlyn Phillipson

The $\mathcal{A}$-discriminant variety, $\nabla_{\mathcal{A}}$, is the irreducible algebraic hypersurface describing all polynomials with singular complex zero set and exponent set contained in $\mathcal{A}$. The tropical $\mathcal{A}$-discriminant is a polyhedral approximation to the image of the log-absolute value applied to $\nabla_{\mathcal{A}}$, and is an important first step toward computationally tractable approximations of discriminant chambers. Such approximations are central in providing results on the topology of real zero sets and faster homotopies preserving the number of real roots. Understanding the real solutions of polynomial equations has applications in numerous disciplines such as robotics and analyzing chemical reactions.

We will present an algorithm, and related quantitative estimates, to fully understand the tropical discriminant for $\mathcal{A} \subset \mathbb{Z}^n$ of cardinality $n + 4$.

11:05 - 11:20 *Inoculation Strategies for Polio: Modeling the Effects of a Growing Population on Public Health Outcomes*
Meredith McCormack-Mager, Wellesley College
Research Mentor: Jay Walton

The World Health Organization has called for global eradication of Polio by 2018, but the disease remains endemic in three countries. Using an expanded SIR model, we take into account the effects of a rapidly growing population on the effectiveness of various vaccination protocols and on the burden of disease in the community. Necessary and sufficient conditions were found for elimination of Polio in such a population in the event of an outbreak.

11:25 - 11:40 *Strong Solution to Smale's 17th Problem for Strongly Sparse Systems*
Paula Burkhardt, Pomona College
Research Mentor: J. Maurice Rojas, Kaitlyn Phillipson

Smale's 17th problem asks whether one can deterministically approximate a single root of a system of polynomials, in polynomial-time on average. The best recent results are probabilistic polynomial-time algorithms, so Smale's 17th Problem has not yet been fully solved. We give a much faster deterministic algorithm for the special case of binomial systems and certain systems of binomials and trinomials. Our approach is also a stepping stone to harder variants of Smale's 17th Problem, such as approximating roots near a query point or approximating a single real root.

11:45 - 12:00 *Binomial Solutions to Smale's 17th Problem and their Application to Chemical Reaction Networks*
Caleb Xavier Bugg, Morehouse College
Research Mentors: J. Maurice Rojas, Kaitlyn Phillipson

In 1998, Stephen Smale proposed a list of eighteen questions for the mathematical community. Smale's 17th problem is concerned with the development of a deterministic algorithm that can approximate a root of a random polynomial system in polynomial-time. This project in its current form provides a positive answer to Smale's 17th problem for binomial systems, and explores a concrete application of the algorithm in chemistry. The main result of this project is an algorithm that approximates a root of an entire binomial system (n variables, n equations) in polynomial-time. The algorithm utilizes matrix exponentiation and the Smith Normal Form of an integer matrix in order convert the system to a simpler system.

Binomial systems are used in a variety of mathematical modeling situations. In particular, there are sufficient, easily verifiable conditions for the expression of a chemical reaction network (CRN) as a binomial system. We utilize our algorithm to solve these systems to determine the steady-state concentrations of the species in CRNs.

1:00 - 1:15 *The Chowla-Selberg Formula for Quartic Abelian CM fields*[1]
Robert Cass, University of Kentucky
Research Mentor: Riad Masri

The Chowla-Selberg formula is a striking identity which relates values of the Dedekind Eta function at quadratic points in the complex upper half-plane to products of values of Euler's Gamma function at rational numbers. We will provide explicit analogues of the Chowla-Selberg formula for quartic Abelian CM fields. This consists of two main parts. First, we implement an algorithm to compute the CM points at which we will evaluate a certain Hilbert modular function which generalizes the Dedekind Eta function. Second, we exhibit families of quartic fields for which we can determine the precise form of the analogue of the product of Gamma values. We will include several examples of our formulas for specific quartic fields.

[1] complex multiplication fields

1:20 - 1:35 *Intervention Model for Malaria*
Taylor McClanahan, University of Arkansas at Little Rock
Research Mentor: Jay Walton

Every year up to about 300 million people are infected by malaria, an infectious disease caused by *Plasmodium* species parasites. Consequently, nearly $660,000$ deaths occur. Infected female *Anophele* mosquitoes transmit the parasite to humans through their salivary glands. Although there are other species that cause malaria, *P. falciparum* is the most dangerous type that infects and is transmitted by humans. Mosquito elimination, avoidance of mosquitoes, sleeping nets, and spraying insecticides are a few methods to retard the spread of malaria. My project focused on the impact of sleeping nets in sub-Saharan countries in Africa. A mathematical model was created and simulated by using eight differential equations. Among a number of results, a set of inequalities were derived that give necessary and sufficient conditions under which the disease would become endemic in a population. These inequalities highlight the role that the use of sleeping nets plays in suppressing or diminishing a malaria outbreak.

1:40 - 1:55 *Zeros of Maass Forms*
Oscar González, University of Puerto Rico Río Piedras Campus
Research Mentor: Matthew P. Young

Maass forms are a certain generalization of modular forms, in which the differential operator condition is slightly relaxed. Here, we study the location of the zeros of the Maass form obtained by applying the Maass level raising operator to the weight k Eisenstein series, and present some conjectures about the location of the zeros of other Maass forms.

2:05 - 2:20 *Representations by Ternary Quadratic Forms*
Edna Jones, Rose Hulman Institute of Technology
Research Mentor: Matthew P. Young

Let $Q(\vec{\mathbf{x}})$ be a positive definite diagonal ternary quadratic form such that $Q(\vec{\mathbf{x}}) = ax^2 + by^2 + cz^2$, where a, b, c are positive integers and $\vec{\mathbf{x}} = (x, y, z)^T$. An integer m is (globally) represented by Q if there exists $\vec{\mathbf{x}} \in \mathbb{Z}^3$ such that $Q(\vec{\mathbf{x}}) = m$. An integer m is locally represented everywhere by Q if for every positive prime integer p and every non-negative integer k there exists $\vec{\mathbf{x}} \in \mathbb{Z}^3$ such that $Q(\vec{\mathbf{x}}) \equiv m \pmod{p^k}$ and there exists $\vec{\mathbf{x}} \in \mathbb{R}^3$ such that $Q(\vec{\mathbf{x}}) = m$.

When is m locally represented everywhere by Q? Are there integers locally represented everywhere by Q but not globally represented by Q? Methods involving quadratic Gauss sums can be used to help answer these questions.

2:25 - 2:40 *Modeling Cyclophosphamide's Effect on Leukocytes*
Matea Alvarado,
Research Mentor: Jay Walton

A dangerous side effect of a chemotherapy drug called cyclophosphamide can contribute to the advancement of cancer treatment. Modeling the toxic effect of cyclophosphamide on leukocytes has implications in assisting oncolytic virotherapy, the use of engineered viruses to combat cancerous cells. The pharmacodynamics of cyclophosphamide and it's metabolites are modeled as well as its direct and indirect effect on leukocytes numbers. This is accomplished by using compartmental ordinary differential equations. The ultimate goal is optimizing the dosage of cyclophosphamide such that the leukocyte population is suppressed while the cyclophosphamide concentration is maintained in a safe range. The viral response of leukocytes must is taken into account as well as the long term effects of the daily dosage. Due to the complexity of these interactions the program Mathematica is used to find the optimal dosing.

2:45 - 3:00 *A Class Structured Mathematical Model For Polio Virus In Nigeria*
Jane Thompson, Williams College
Research Mentor: Jay Walton

Polio virus transmission rates are investigated using a mathematical model of ordinary differential equations. Transmission is restricted to the fecal-oral route in this model. The model is used to collect data for the values of its unknown parameters. The simulations are repeated using different probability distributions and the likelihood of total eradication is derived in each case. The results suggest that lower environmental virus concentrations can lead to eradication when the length of the infectious period is distributed exponentially. Increased environmental clean up efforts are more effective than increased vaccinations when gamma follows a Gamma Distribution.

3:10 - 3:25 *Effective Non-vanishing of Class Group L-functions for Biquadratic CM Fields (part 1)*
Katy Weber, State University of New York, Geneseo
Research Mentor: Riad Masri

We introduce class group L-functions and the mathematical objects that constitute them, as well as their connection to Eisenstein series. We will then state a result regarding the non-vanishing of L-functions attached to class group characters of biquadratic CM fields of the form $\mathbb{Q}(\sqrt{d_1}, \sqrt{d_2})$, where $d_1 > 0$ and $d_2 < 0$ are relatively prime square-free integers.

3:30 - 3:45 *Effective Non-vanishing of Class Group L-functions for Biquadratic CM Fields (part 2)*
Emily Peirce, Baylor University
Research Mentor: Riad Masri

We outline a proof that, given relatively prime square-free integers $d_1 > 0$ and $d_2 < 0$ such that $d_1 \geq 260,868$ and $|d_2| \geq \max\{C_1(d_1), C_2(d_1)\}$ for some explicit positive constants C_1 and C_2, there exists at least one class group character χ of the biquadratic CM field $\mathbb{Q}(\sqrt{d_1}, \sqrt{d_2})$ such that the L-function $L(\chi, s)$ attached to this character is non-vanishing at $s = \frac{1}{2}$.