On Classification of Low-Dimensional Irreducible Representations of B_5

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Abstract

The design for a topological quantum computer is based on anyon braiding. It uses topology to protect quantum information against decoherence. We may model the space-time trajectory of a system of n anyons with the n-strand braid group B_n. Storing and manipulating information in the representation spaces of B_n is the foundation of Topological Quantum Computation, thus understanding the representations of these braid groups is an important problem. In this talk, we present results on the classication of the unitarizability of low-dimensional irreducible representations of B_5. Using symbolic MatLab, we have determined that the Hecke algebra and reduced-extended Burau representations of B_5 are not unitarizable. The methods developed in this paper may be easily adapted to any given representation for B_n of a given n.

1 Introduction & Background

An exciting development in the field of computer science is the theoretical possibility of quantum computing (QC).

The instability of quantum wavestates used in QC poses a challenge to building a functioning quantum computer. These states decay very rapidly, resulting in a loss of quantum information called decoherence.

A proposed solution to the problem of decoherence is topological quantum computation (TQC). A topological quantum computer uses anyons as quantum bits (qubits). Anyons are quasi-particles whose space-time trajectories form the strands of the braid group B_n.

The braid group B_n is the group which contains all possible braidings of n-strands. The group B_n is generated by the elementary generator σ_i for $i \in \{1, ..., n-1\}$ which twists the ith and $i+1$th strands via the right-handed convention.

Definition 1.1. The braid group B_n is defined by the following generators and relations, $B_n = \langle \sigma_1, \sigma_2, ..., \sigma_n - 1 | \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$ for all $i \in \{1, ..., n-1\}$ and $\sigma_i \sigma_j = \sigma_j \sigma_i$ for all $|i-j| \neq 1\rangle$

Topological quantum computing is based on using the representation spaces of B_n to store and manipulate information [1].

Definition 1.2. A **representation** of a group G is a pair (ρ, V), where V is a d-dimensional vector space and ρ is a group homomorphism from G to the collection of $d \times d$ invertible matrices over $\mathbb{C}$.

We have a particular interest in irreducible representations of B_n.

Definition 1.3. A representation is called **irreducible** if it contains no invariant subspaces.

Definition 1.4. A subspace W is called **invariant** if $\rho(g)(W) \subseteq W$ for all $g \in G$.

Many representations may be built from irreducible representations. Then we may think of an irreducible representation as the fundamental building block of larger-more complex representations.

The quantum wavestate of an anyon may be represented as a vector element of a complex Hilbert space. We may manipulate this fundamental unit of quantum information by applying a unitary matrix. Understanding which irreducible representations of the braid group are unitarizable is thus fundamentally important to TQC.

Definition 1.5. A representation ρ is **unitarizable** if there exists a Hermitian inner product $\langle \cdot | \cdot \rangle_A$ such that $\langle \rho(g)v | \rho(g)w \rangle_A = \langle v|w \rangle_A$ for all $g \in G$ and for all $v, w \in V$.

Let's take a brief moment to develop some intuition for what it means for a matrix to be unitary.

Example 1.1. Recall that we may recover the usual notion of the length of a vector v from the standard inner product via $\langle v|v \rangle$. Let $\rho(g)$ be a unitary matrix, then it follows that $\langle \rho(g)v|\rho(g)v \rangle = \langle v|v \rangle$. In other words, applying a unitary matrix to a vector does not change the vector's length. Unitary matrices in this context would correspond to rotations of the vector v through some angle.

Next we introduce useful tools for the main analysis.

Definition 1.6. Let A be a matrix. We define the **adjoint** of the matrix $\rho(g)$ via $\rho(g)^* = A^{-1}\rho(g)^\dagger A$, where $\dagger$ denotes the complex conjugate transpose.

Theorem 1.1. Let $v, w \in \mathbb{C}^d$ and $\rho(g) \in GL_d(\mathbb{C})$. We have $\langle \rho(g)v|\rho(g)w \rangle_A = \langle v|w \rangle_A$ if and only if there exists a matrix A such that $\rho(g)\rho(g)^* = I$.

We will make use of the Burau representation $\beta_n(t) : B_n \to GL_{n-1}(\mathbb{C}[t^{\pm 1}])$ which is given in [2] by,

$$\beta_n(t)(\sigma_1) = \left[\begin{array}{cc|c} -t & 0 & \multirow{2}{*}{0} \\ -1 & 1 & \\ \hline \multicolumn{2}{c|}{0} & I_{n-3} \end{array}\right], \beta_n(t)(\sigma_i) = \left[\begin{array}{c|ccc|c} I_{i-2} & & 0 & & 0 \\ \hline & 1 & -t & 0 & \\ 0 & 0 & -t & 0 & 0 \\ & 0 & -1 & 1 & \\ \hline 0 & & 0 & & I_{n-i-2} \end{array}\right], \beta(t)(\sigma_{n-1}) = \left[\begin{array}{c|cc} I_{n-3} & \multicolumn{2}{c}{0} \\ \hline \multirow{2}{*}{0} & 1 & -t \\ & 0 & -t \end{array}\right]$$

It should be noted that $t \neq 0$, or else this would not be a representation. We will also make use of the standard representation $s_n(t) : B_n \to GL_n(\mathbb{C}[t^{\pm 1}])$ defined by

$$s(t)(\sigma_i) = \left[\begin{array}{c|c|c} I_{i-1} & 0 & 0 \\ \hline 0 & \begin{matrix} 0 & t \\ 1 & 0 \end{matrix} & 0 \\ \hline 0 & 0 & I_{n-(i-1)} \end{array}\right]$$

Theorem 1.2. The standard representation is unitarizable when $t\bar{t} = 1$.

Proof. Let $A = I$, and assume $t\bar{t} = 1$, then

$$\begin{aligned} s(t)(\sigma_i)s(t)(\sigma_i)^* &= s(t)(\sigma_i)A^{-1}s(t)(\sigma_i)^\dagger A \\ &= s(t)(\sigma_i)I^{-1}s(t)(\sigma_i)^\dagger I \\ &= s(t)(\sigma_i)s(t)(\sigma_i)^\dagger \\ &= \left[\begin{array}{c|c|c} I_{i-1} & 0 & 0 \\ \hline 0 & \begin{matrix} 0 & t \\ 1 & 0 \end{matrix} & 0 \\ \hline 0 & 0 & I_{n-(i-1)} \end{array}\right] \left[\begin{array}{c|c|c} I_{i-1} & 0 & 0 \\ \hline 0 & \begin{matrix} 0 & 1 \\ \bar{t} & 0 \end{matrix} & 0 \\ \hline 0 & 0 & I_{n-(i-1)} \end{array}\right] \\ &= \left[\begin{array}{c|c|c} I_{i-1} & 0 & 0 \\ \hline 0 & \begin{matrix} t\bar{t} & 0 \\ 0 & 1 \end{matrix} & 0 \\ \hline 0 & 0 & I_{n-(i-1)} \end{array}\right] \\ &= I \end{aligned}$$

□

Now we are ready to develop the main result.

2 Main Results

Formanek et al. showed that all irreducible representations of dimension $d \leq n$ take the following form: $\chi(c) \otimes \rho(t)$, where t and c are parameters, $\chi(c)$ is a one-dimensional representation defined by $\chi(c)(\sigma_i) = c$, and $\rho(t)$ is one of a finite list of given representations [2].

Then in order to classify the unitarizability of representations of B_5 with $d \leq 5$, we need to check each $\rho(t)$ provided by Formanek et al. We have developed a process by which unitarizability may be assessed, which is outlined bellow.

2.1 Set Up

Let $\chi(c)$ be the one-dimensional representation given by $\chi(c)(\sigma_i) = c \in \mathbb{C}^*$, and let $\rho(t)$ be a representation of the braid group B_5 such that $\tilde{\rho}(\sigma_i) = (\chi(c) \otimes \rho(t))(\sigma_i)$ has dimension

$d \leq 5$. Then $\tilde{\rho}$ is unitarizable if and only if there exists a matrix A such that

$$\begin{aligned} I &= \tilde{\rho}(\sigma_i)(\tilde{\rho}(\sigma_i))^* \\ &= \tilde{\rho}(\sigma_i)A^{-1}\tilde{\rho}(\sigma_i)^\dagger A \end{aligned}$$

This is a matrix equation which we solve using symbolic MatLab. In order to accomplish this, we will first simplify this expression by writing it only in terms of A, not in terms of A and A^{-1}. We will then express the equation in terms of $\rho(t)$ and c.

We begin by multiplying from the right by A^{-1}, then multiplying from the left by A, we have $I = A\tilde{\rho}A^{-1}\tilde{\rho}^\dagger$. If we multiply from the right by $(\tilde{\rho}^\dagger)^{-1}A$, then we see that $(\tilde{\rho}^\dagger)^{-1}A = A\tilde{\rho}$. Subtracting $(\tilde{\rho}^\dagger)^{-1}A$ from both sides gives

$$0 = A\tilde{\rho}(\sigma_i) - (\tilde{\rho}(\sigma_i)^\dagger)^{-1}A \tag{1}$$

By substituting $\tilde{\rho}(\sigma_i) = (\chi(c) \otimes \rho(t))(\sigma_i)$ into (1), we see that

$$\begin{aligned} 0 &= A\tilde{\rho}(\sigma_i) - (\tilde{\rho}(\sigma_i)^\dagger)^{-1}A \\ &= A(\chi(c) \otimes \rho(t))(\sigma_i) - ((\chi(c) \otimes \rho(t))(\sigma_i)^\dagger)^{-1}A \end{aligned}$$

Using the fact that $(\chi(c) \otimes \rho(t))(\sigma_i) = c\rho(t)(\sigma_i)$, we have

$$\begin{aligned} &= A(\chi(c) \otimes \rho(t))(\sigma_i) - ((\chi(c) \otimes \rho(t))(\sigma_i)^\dagger)^{-1}A \\ &= A(c\rho(t)(\sigma_i)) - ((c\rho(t)(\sigma_i))^\dagger)^{-1}A \\ &= c(A\rho(t)(\sigma_i)) - \frac{1}{\bar{c}}((\rho(t)(\sigma_i))^\dagger)^{-1}A \end{aligned}$$

We then multiply both sides of $0 = c(A\rho(t)(\sigma_i)) - \frac{1}{\bar{c}}((\rho(t)(\sigma_i))^\dagger)^{-1}A$ by $\bar{c}$, which gives.

$$0 = c\bar{c}(A\rho(t)(\sigma_i)) - ((\rho(t)(\sigma_i))^\dagger)^{-1}A \tag{2}$$

This equation may now be solved symbolically given $\rho(t)(\sigma_i)$, thereby determining the unitarizability of $\tilde{\rho}$. It is clear that if c is on the unit circle then $\tilde{\rho}$ is unitarizable if and only if $\rho(t)$ is unitarizable. However, the existence of some non-unitarizable $\rho(t)$ and appropriate choice of c such that $\tilde{\rho}$ is unitarizable is not known.

We now outline our analysis of the unitarizability of $\tilde{\rho}$ given $\rho(t)$.

2.2 Solution

Let $\rho(t_1)_1 = H_5(t_1)$ and $\rho(t_2)_2 = \hat{\beta}_5(t_2)$ be representations of B_5 with dimensions $d_1 = 5$ and $d_2 = 3$ respectively. Let H be the Hecke algebra representation, given in [2] by

$$H_5(t_1)(\sigma_1) = \begin{bmatrix} 1 & 0 & 0 & 0 & -t_1 \\ 0 & -t_1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -t_1 \end{bmatrix}, \quad H_5(t_1)(\sigma_2) = \begin{bmatrix} -t_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -t_1 & 0 & 0 \\ 0 & 0 & -t_1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5(t_1)(\sigma_3) = \begin{bmatrix} 1 & 0 & 0 & -t_1 & 0 \\ 0 & -t_1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -t_1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}, \quad H_5(t_1)(\sigma_4) = \begin{bmatrix} -t_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -t_1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -t_1 \end{bmatrix}$$

Note both that this representation is irreducible provided t_1 is not a root of $(x^2+x+1)(x^2+1)$, and that if $t_1 = 0$ then our matrices no longer are invertible and as such do not define a representation.
Let t_2 be a root of $x^4+x^3+x^2+x+1$, then $\hat{\beta}_5(t_2)$ is the irreducible representation defined in [2] by $\hat{\beta}_5(t_2)(\sigma_i) = \beta_4(t_2)(\sigma_i)$ for $i \in \{1,2,3\}$ and $\hat{\beta}_5(t_2)(\sigma_4) = I - PQ$, where

$$P = \begin{pmatrix} 0 \\ 0 \\ t_2 \end{pmatrix}, \quad Q = t_2(1, -(1+t_2), (1+t_2+t_2^2)).$$

We may evaluate the unitarizability of $\tilde{\rho}_j = \chi(c) \otimes \rho(t_j)_j$ for $j = 1,2$ using the MatLab code designed to evaluate (1), which may be found in the appendix. This code is able to directly check the unitarizability of a representation given a value of c and t_j. Otherwise the code outputs a coefficient matrix that way may use to solve for the entries a_{kl} of A in terms of t_j and c.
To simplify the analysis, we will show that our representations are not unitarizable by showing that the matrix A has a zero row.

2.2.1 The Hecke Algebra Representation $H_5(t_1)$

Proof. Assume t_1 is not a root of $(x^2+x+1)(x^2+1)$. The output of our code for $j = 1$ is a 100×25 coefficient matrix with each row is given by an entry of the equation matrix given by (1) for all σ_i. We evaluate the unitarizability of $\tilde{\rho}_1$ via the following equations recovered from the rows of the coefficient matrix:

$$(c\bar{c}-1)a_{41} = 0, \tag{3}$$
$$(c\bar{c}-1)a_{42} = 0, \tag{4}$$
$$(c\bar{c}-1)a_{43} = 0, \tag{5}$$
$$(c\bar{c}-1)a_{44} = 0, \tag{6}$$
$$(c\bar{c}-1)a_{45} = 0 \tag{7}$$

From equations (3-7) we see that either $c\bar{c} = 1$ or $a_{41} = a_{42} = a_{43} = a_{44} = a_{45} = 0$. If our representation is to be unitarizable, A can not have any zero rows, so we conclude that $c\bar{c} = 1$. Then

$$0 = c\bar{c}(AH_5(t_1)(\sigma_i)) - ((H_5(t_1)(\sigma_i))^\dagger)^{-1}A \tag{8}$$
$$= (AH_5(t_1)(\sigma_i)) - ((H_5(t_1)(\sigma_i))^\dagger)^{-1}A \tag{9}$$

Thus, $\tilde{\rho}_1 = \chi(c) \otimes H_5(t_1)$ is unitarizable if and only if $H_5(t_1)$ is unitarizable.
Using $c\bar{c} = 1$, we simplify our original coefficient matrix, and produce the following equations:

$$-t_1 a_{22} = 0 \tag{10}$$
$$-\bar{t_1} a_{23} = 0 \tag{11}$$
$$-\bar{t_1} a_{24} = 0 \tag{12}$$
$$a_{24} + (\frac{1}{\bar{t_1}} + 1)a_{34} = 0 \tag{13}$$
$$a_{25} + (\frac{1}{\bar{t_1}} + 1)a_{35} = 0 \tag{14}$$
$$-t_1 a_{31} - (t_1 + 1)a_{34} = 0 \tag{15}$$
$$-t_1 a_{31} - (t_1 + 1)a_{35} = 0 \tag{16}$$
$$-(t_1 + 1)a_{21} - a_{24} - a_{25} = 0 \tag{17}$$

From these equations we will show that A must have a zero row.
Since $t_1 \neq 0$, it follows from $(10-12)$ that $a_{22} = a_{23} = a_{24} = 0$. Since $a_{24} = 0$, we obtain $(\frac{1}{\bar{t_1}} + 1)a_{34} = 0$ from (13). Then either $t_1 = -1$ or $a_{34} = 0$. Since our code can easily check the unitarizability of a representation given c and t_1, we simply input $t_1 = -1$ and see that it is not unitarizable.
Thus we may assume $t_1 \neq -1$, and conclude that $a_{34} = 0$. By applying this fact to (15), we see that $-t_1 a_{31} = 0$. We then apply this result to (16), and conclude that $a_{35} = 0$. From this fact we get $a_{25} = 0$ from (14). Finally, since $a_{24} = a_{25} = 0$, and since $t_1 \neq -1$, we see that $a_{21} = 0$ from (17). Thus $a_{21} = a_{22} = a_{23} = a_{24} = a_{25} = 0$.
Therefore A has a zero row, and we see that $H_5(t_1)$, and thus $\chi(c) \otimes H_5(t_1)$, is not unitarizable.
□

2.2.2 The Reduced-Extended Burau Representation $\hat{\beta}_5$

Proof. Assume t_2 is a root of $f(t_2) = x^4 + x^3 + x^2 + x + 1$. For $j = 2$ our code provides a 36×9 coefficient matrix, where each row corresponds to an entry of the equation matrix given by (1) for each σ_i. From this matrix, we evaluate the unitarizability of $\chi(c) \otimes \hat{\beta}(t_2)$ via the following equations:

$$(c\bar{c} - 1)a_{11} = 0, \tag{18}$$
$$(c\bar{c} - 1)a_{12} = 0, \tag{19}$$
$$(c\bar{c} - 1)a_{13} = 0, \tag{20}$$

Thus either A has a zero row or $c\bar{c} = 1$. If $c\bar{c} = 1$, then we need only check the unitarizability of $\hat{\beta}(t_2)$. Using this fact, we simplify our coefficient matrix and evaluate the following equations:

$$\left(\frac{1}{\bar{t_2}^4+\bar{t_2}^3+\bar{t_2}^2-1}+1\right)a_{31}-t_2^2a_{33}=0 \tag{21}$$

$$\left(\frac{1}{\bar{t_2}^4+\bar{t_2}^3+\bar{t_2}^2-1}+1\right)a_{32}+(t_2^3+t_2^2)a_{33}=0 \tag{22}$$

$$\left(\frac{1}{\bar{t_2}^4+\bar{t_2}^3+\bar{t_2}^2-1}-(t_2^4+t_2^3+t_2^2)+1\right)a_{33}=0 \tag{23}$$

From (23), we have either $a_{33}=0$ or $\frac{1}{\bar{t_2}^4+\bar{t_2}^3+\bar{t_2}^2-1}=(t_2^4+t_2^3+t_2^2)-1$. Solving this for t_2 we have $t_2=0,-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i,-1.2409,0.8718$, but since none of these are roots of $f(t_2)$ we conclude that $a_{33}=0$.
From these assumptions, it quickly follows from (21) and (22) that $a_{31}=a_{32}=a_{33}=0$. Thus A has a zero row, and we conclude that $\hat{\beta}(t_2)$ is not unitarizable. Therefore $\chi(c)\otimes\hat{\beta}(t_2)$ is not unitarizable. □

3 Discussion

3.1 Summary & Next Steps

Using a combination of numerical and symbolic MatLab, we have successfully determined the unitarizability of $H_5(t_1)$ and $\hat{\beta}(t_2)$. Our methods hinge on a simple script and are easily adapted to other representations. We have thus demonstrated how any similar representation's unitarizability may be evaluated.
Since Formanek et al. showed that there are only a finite number of representation classes up to equivalence-it is thus possible to use our approach to fully classify the unitarizability of the representations of the braid group with a given number of strands.
Using these methods, collaboration with another researcher has resulted in a full classification of the unitarizable representations of B_5 with $d\leq 5$, provided bellow.

1. For $d=1$, all unitarizable representations are one-dimensional and of the form $\chi(c)$ where $c\bar{c}=1$.

2. For $d=2$, there are no irreducible unitary representations.

3. For $d=3$, there are no irreducible unitary representations.

4. For $d=4$, we have unitarizable representations of the Burau type $\chi(c)\otimes\beta(t)$, when $c\bar{c}=1$ and the Burau representation is unitary. The unitarizability of the Burau is well-understood and given in [3].

5. For $d=5$, we have unitarizable representations of standard type $\chi(z)\otimes s(t)$.

For our next steps, we will check the unitarizability for representations of B_4, B_6, B_7, and B_8. Once this is done, then a full classification of unitarizable representations of B_n will be more easily attainable since after $n = 8$ we only have the Burau, reduced-extended Burau and standard representation types.

Since the unitarizability of the Burau and the standard representations is already known, we only need to analyze the unitarizability of the reduced-extended Burau.

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Appendix

The MatLab script used for this project is provided bellow:

```
%Hecke algebra/burau hat reps of B5
syms c
t = 2
%hecke
XH = sym('x', 5);

H_1 = [1,0,0,0,-1;
       0,-1,0,0,0;
       0,-1,1,0,0;
       0,-1,0,1,0;
       0,0,0,0,-1];


H_2 = [-1,0,0,0,0;
       0,1,-1,0,0;
       0,0,-1,0,0;
       -1,0,0,1,0;
       -1,0,0,0,1];

H_3 = [1,0,0,-1,0;
       0,-1,0,0,0;
       0,-1,1,0,0;
```

```
       0,0,0,-1,0;
       0,-1,0,0,1];

H_4 = [-t,0,0,0,0;
       0,1,0,0,-t;
       -1,0,1,0,0;
       -1,0,0,1,0;
       -1,0,0,0,1];

%burau
XB = sym('x', 3);

Bh_1 = [-t,0,0;
        -1,1,0;
         0,0,1];
Bh_2 = [1,-t,0;
        0,-t,0;
        0,-1,1];
Bh_3 = [1,0,0;
       0,1,-t;
       0,0,-t];
Bh_4 = eye(3) - ([0;0;t]*[t,-(t+t^2),t+t^2+t^3]);


%Need empty vectors to fill with entries of Yi
%Burau hat
VB1 = [];
VB2 = [];
VB3 = [];
VB4 = [];
%hecke
VH1 = [];
VH2 = [];
VH3 = [];
VH4 = [];
%variable vector
%burau hat
xb = [];
%hecke
xh = [];
%X has to satisfy the following equation matrices
%hecke
YH1 = (c*conj(c))*XH*H_1 - (H_1^(-1)')*XH == 0;
YH2 = (c*conj(c))*XH*H_2 - (H_2^(-1)')*XH == 0;
YH3 = (c*conj(c))*XH*H_3 - (H_3^(-1)')*XH == 0;
```

```
YH4 = (c*conj(c))*XH*H_4 - (H_4^(-1)')*XH == 0;
%burau
YB1 = (c*conj(c))*XB*Bh_1 - (Bh_1^(-1)')*XB == 0;
YB2 = (c*conj(c))*XB*Bh_2 - (Bh_2^(-1)')*XB == 0;
YB3 = (c*conj(c))*XB*Bh_3 - (Bh_3^(-1)')*XB == 0;
YB4 = (c*conj(c))*XB*Bh_4 - (Bh_4^(-1)')*XB == 0;
%This puts the above equation matrices in vector form
%hecke
for i= 1:5
    for j = 1:5
        VH1 = [VH1 YH1(i,j)];
        VH2 = [VH2 YH2(i,j)];
        VH3 = [VH3 YH3(i,j)];
        VH4 = [VH4 YH4(i,j)];
        xh = [xh XH(i,j)];
    end
end
%burau
for i= 1:3
    for j = 1:3
        VB1 = [VB1 Y1(i,j)];
        VB2 = [VB2 Y2(i,j)];
        VB3 = [VB3 Y3(i,j)];
        VB4 = [VB4 Y4(i,j)];
        xb = [xb X(i,j)];
    end
end
%master equation vector
%hecke
VH = [VH1 VH2 VH3 VH4];
%burau
VB = [VB1 VB2 VB3 VB4];
%convert all equations from equation matrices Yi into a single coefficient
%matrix
%hecke
MH = equationsToMatrix(VH,xh);
%burau
MB = equationsToMatrix(VB,xb);
%reduced row echelon form of above coefficient matrix
SH = rref(MH);

SB = rref(MB);
%if t was numerical, we need only check S for the solution, if the only
%solution is the zero matrix, the rep is not unitarizable
```

```
%variable entries of X are in the following order:
%[ x1_1, x1_2, x1_3, x1_4, x1_5, x2_1, x2_2, x2_3, x2_4, x2_5, x3_1, x3_2, x3_3, x3_4, x
%x(i) is the variable corresponding to the ith columb of S
```

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