2017 Texas A&M REU Miniconference

July 17–18, Blocker Building, Room 163 (Mon.) and 220 (Tues.)

SCHEDULE

MON., July 17

08:30–08:55	Breakfast in Blocker 246	
09:00–09:20	Max Intersection-Complete Codes and the Neural Ideal	Molly Hoch
09:25–09:45	On Classification of the Unitarizability of Low-Dimensional Irreducible Representations of $B5$	Étude O'Neel-Judy
09:50–10:20	Nonvanishing of Hecke series and ℓ-torsion in class groups	Alex Mathers, Maria Ross, and Arianna Iannuzzi
10:25–10:35	Break!	
10:40–11:00	Classification of Unitarizable Low Dimensional Representations of $B5$	Paul Vienhage
11:05–11:25	Inductively Pierced Codes and Toric Ideals	Samuel Muthiah
11:30–11:50	Convex Codes and Minimal Embedding Dimensions	Megan Franke
12:00–13:00	Lunch in Blocker 246	

TUES., July 18

08:30–08:55	Breakfast in Blocker 246	
09:00–09:20	On classification of (weakly integral) modular categories by dimension	Katie Lee
09:25–09:45	QSSA and Solvability	Mark Sweeney
09:50–10:10	Oscillations in Michaelis-Menten Systems	Ray Tung
10:15–10:25	Break!	
10:30–10:50	Classifying Strictly Weakly Integral Modular Categories of Dimension 16p	Elena Amparo
10:55–11:10	Zeros of Newform Eisenstein Series on $\Gamma_0(N)$ (part 1)	Thomas Brazelton
11:15–11:30	Zeros of Newform Eisenstein Series on $\Gamma_0(N)$ (part 2)	Victoria Jakicic
12:00–13:00	Lunch in Blocker 246	

As always, we gratefully acknowledge the support of our Department Head Emil Straube, Associate Department Head Paulo Lima-Filho, Undergraduate Program Assistant Donna Hoffman, and the support of the National Science Foundation through REU grant DMS-1460766.

ABSTRACTS

(In order of appearance)

Max Intersection-Complete Codes and the Neural Ideal

by Molly Hoch

Place cells represent certain animals' location relative to their surrounding environment. From the receptive fields of these cells, we build neural codes. There is particular biological interest in studying which neural codes can be represented by convex receptive fields. One such type of convex code is the max intersection-complete code. In order to better understand neural codes, we associate to each an ideal, called the neural ideal. Here we study the canonical form of the neural ideal in the interest of finding a signature for max intersection-complete codes. We provide an algorithm for determining maximal codewords given a canonical form, and provide sufficient conditions for a code to be non-convex based on its canonical form.

On Classification of the Unitarizability of Low-Dimensional Irreducible Representations of $B5$

by Étude O'Neel-Judy

The design for a topological quantum computer is based on anyon braiding. It uses topology to protect quantum information against decoherence. We may model the space-time trajectory of a system of n anyons with the n-strand braid group Bn. Storing and manipulating information in the representation spaces of Bn is the foundation of Topological Quantum Computation, thus understanding the representations of these braid groups is an important problem. In this talk, we present results on the classification of the unitarizability of low-dimensional irreducible representations of Bn, and especially of $B5$.

Nonvanishing of Hecke series and ℓ-torsion in class groups

by Alex Mathers, Maria Ross, and Arianna Iannuzzi

We give an asymptotic formula with a power-saving error term for the density of imaginary quadratic fields satisfying a certain lower bound on the number of canonical Hecke L-series with nonvanishing central value. Our work combines an effective nonvanishing theorem for Hecke L-series with recent work of Ellenberg, Pierce and Wood on bounds of ℓ-torsion in class groups.

Classification of Unitarizable Low Dimensional Representations of $B5$

by Paul Vienhage

The braid group on five strands, $B5$, is a structure which represents the twisting of five strings into braids, where the group operation which is concatenation of braids. This group is used to study the motion of 5 points in a disk. A representation describes the action of the group on certain complex vector space by identifying it with automorphisms of the vector space via a group morphism. We will present results which classify which of the representations of $B5$ are unitarizable, building on previous results which classify the irreducible representations. This topic has direct applications in the mathematics of topological quantum computers, which perform computation by using and braiding anyons. Our results are useful in understanding these types of computers.

Inductively Pierced Codes and Toric Ideals

by Samuel Muthiah

Neural codes are binary codes in $\{0,1\}^n$; here we focus on the ones which represent the firing patterns of a type of neurons called place cells. There is much interest in determining which neural codes can be realized by a collection of convex sets. However, drawing these convex sets, particularly as the number of neurons in a code increases, can be very difficult. It has been shown that an algorithm for drawing Euler diagrams can be used to draw a class of codes that are said to be k-inductively pierced. We use two ideals, the neural ideal and the toric ideal, to show sufficient conditions for a code to be 1- or 2-inductively pierced.

Convex Codes and Minimal Embedding Dimensions

by Megan Franke

Place cells are neurons found in some mammals that fire based on the animals location in their environment. These place cells fire in approximately convex regions called receptive fields which are subsets of a Euclidean space. From the intersections of these receptive fields, a corresponding code is extracted, leading us to ask: is any code realizable by convex sets in a Euclidean space? We first provide a construction of a convex realization of an arbitrary code in $\mathbb{R}^{d-1}$ where d is the number of nonempty codewords in our code. We then explore the relationship between a code and the minimal dimension it is realizable in, called the minimal embedding dimension, and conclude that there is no upper bound of the minimal embedding dimensions of all codes.

On classification of (weakly integral) modular categories by dimension
by Katie Lee
In this talk, we discuss the classification of modular categories by dimension. Motivation to classify these categories comes from their importance in various fields of mathematics, including topological quantum field theory, conformal field theory, representation theory of quantum groups, vertex operator algebras and applications in physics. Due to the difficulties of classifying these modular categories in general, we start by looking at modular categories of specific dimensions. In particular we consider modular categories of dimension 2^n and strictly weakly integral modular categories of dimension $4q^2$.

QSSA and Solvability
by Mark Sweeney
The quasi-steady-state assumption (QSSA) is a widely used approximation in chemistry and chemical engineering to simplify reaction mechanisms. The key step in the method requires a solution by radicals of a system of multivariate polynomials. Pantea et al. (2014) showed that there exist mechanisms for which the associated polynomials are not solvable by radicals. We present a small class of chemical reaction networks for which solvability is guaranteed, and extend this to the positive real steady states of larger networks. Several examples applying these results will be discussed.

Oscillations in Michaelis-Menten Systems
by Ray Tung
Oscillations play a major role in a number of biological systems, the most notable example in biochemistry being circadian clocks. In this talk we focus on the existence of oscillations within a 2-site phosphorylation system. Previously, it has been shown by Wang and Sontag that the Michaelis-Menten (MM) approximation of the distributive and sequential 2-site phosphorylation system lacks oscillations using monotone systems theory. However, biological systems are generally not purely distributive; there is generally some processive behavior as well. This talk expands on the methods of Bozeman and Morales to find conditions for the existence of oscillations in the MM approximation of a general 2-site phosphorylation system. By studying the MM approximation, light may be shed on the existence of oscillations in the original system.

Classifying Strictly Weakly Integral Modular Categories of Dimension $16p$
by Elena Amparo
Modular categories have an algebraic structure which makes them easier to study. A complete classification of modular categories is motivated by applications to physics, where modular categories correspond to $(2+1)$-dimensional topological quantum field theories and are used in models of topological quantum computation. We classify strictly weakly integral modular categories of dimension 16p according to the dimensions of their simple objects.

Zeros of Newform Eisenstein Series on $\Gamma_0(N)$
by Thomas Brazelton and Victoria Jakicic
Given primitive Dirichlet characters χ_1 and χ_2 modulo q_1 and q_2 respectively, we examine the newform Eisenstein series $E_{\chi_1,\chi_2,k}(z)$ of weight k on $\Gamma_0(q_1q_2)$. We determine the location and distribution of a significant fraction of the zeros of these Eisenstein series as the weight k is sufficiently large. Additionally, we display partial evidence that the zeros of these Eisenstein series equidistribute as q_1q_2 tends to infinity.